Protecting Against Fault Injection Attacks: from CRT-RSA to all Asymmetric Cryptography

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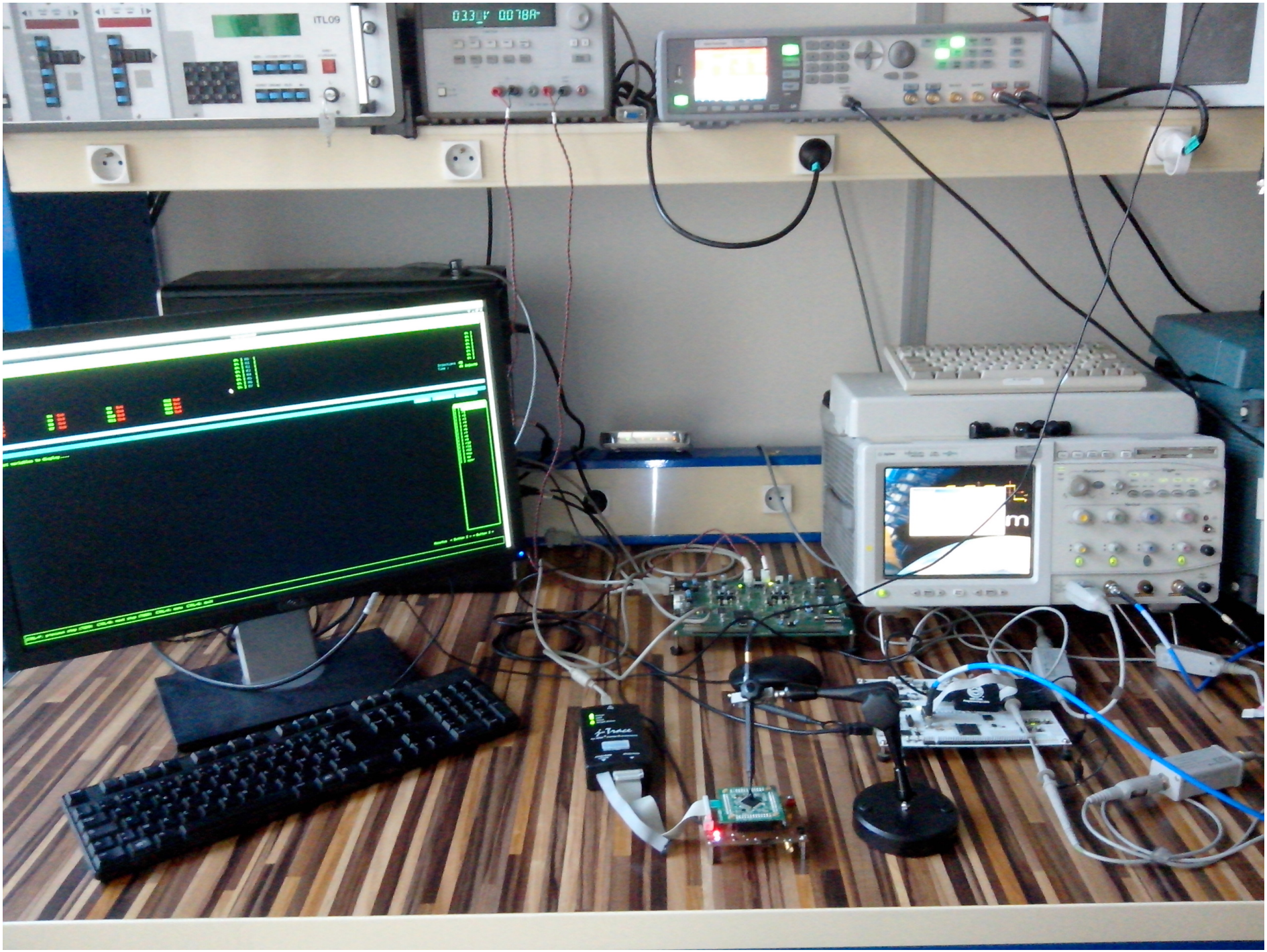
ITL09

RSA (*Rivest, Shamir, Adleman*) — **Definition**

RSA [RSA78] is an algorithm for public key cryptography. It can be used as both an encryption and a signature algorithm.

- Let M be the message, (N, e) the public key, and (N, d) the private key such that $d \cdot e \equiv 1 \mod \varphi(N)$.
- The signature S is computed by $S \equiv M^d \mod N$.
- The signature can be verified by checking that $M \equiv S^e \mod N$.

CRT (*Chinese Remainder Theorem*) — Definition

CRT-RSA [Koç94] is an optimization of the RSA computation which allows a fourfold speedup.

- Let p and q be the primes from the key generation ($N = p \cdot q$).
- These values are pre-computed (considered part of the private key):
  - $d_p \doteq d \mod (p-1)$
  - $d_q \doteq d \mod (q-1)$
  - $i_q \doteq q^{-1} \mod p$
- S is then computed as follows:
  - $S_p = M^{d_p} \mod p$
  - $S_q = M^{d_q} \mod q$
  - $S = S_q + q \cdot (i_q \cdot (S_p - S_q) \mod p)$
    (recombination method of [Gar65]).

BellCoRe (*Bell Communications Research*) — **Definition**

The BellCoRe attack [BDL97] consists in revealing the secret primes p and q by faulting the computation. It is very powerful as it works even with very random faulting.

- The intermediate variable S_p (resp. S_q) is faulted as $\widehat{S_p}$ (resp. $\widehat{S_q}$).
- The attacker thus gets an erroneous signature $\widehat{S}$.
- The attacker can recover p (resp. q) as $\gcd(N, S - \widehat{S})$.

Why does it Work?

- For all integer x, $\gcd(N, x)$ can only take 4 values:
  - 1, if N and x are co-prime,
  - p, if x is a multiple of p,
  - q, if x is a multiple of q,
  - N, if x is a multiple of both p and q, i.e., of N.

Why does it Work?

- If S_p is faulted (i.e., replaced by $\widehat{S_p} \neq S_p$):
  - $S - \widehat{S} = q \cdot \Big((i_q \cdot (S_p - S_q) \mod p) - (i_q \cdot (\widehat{S_p} - S_q) \mod p) \Big)$

  $\Rightarrow \gcd(N, S - \widehat{S}) = q$

Why does it Work?

- If S_q is faulted (i.e., replaced by $\widehat{S_q} \neq S_q$):
  - $S - \widehat{S} \equiv (S_q - \widehat{S_q}) - (q \mod p) \cdot i_q \cdot (S_q - \widehat{S_q}) \equiv 0 \mod p$
    (because $(q \mod p) \cdot i_q \equiv 1 \mod p$)
  - $\Rightarrow \gcd(N, S - \widehat{S}) = p$

Several protections against the BellCoRe attacks have been proposed.

Some of them are given below:

- Obvious countermeasures: no CRT, or with signature verification;
- Shamir [Sha99];
- Aumüller et al. [ABF+02];
- Vigilant, original [Vig08] and with some corrections by Coron et al. [CGM+10];
- Rivain [Riv09];
- Blömer et al. [BOS03];
- Kim et al. [KKHH11].
- ...

- The goal is making sure countermeasures are trustworthy.
- We want to cover a very general attacker model.
- We want our proof to apply to any implementation that is a refinement of the abstract algorithm.

$\Rightarrow$ We want our tool to offer a full fault coverage of CRT-RSA algorithm, thereby keeping the proof valid even if the code is transformed (e.g., optimized, compiled, partitioned in software/hardware, or equipped with dedicated countermeasures).

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Attacker Model

- An attacker can request a CRT-RSA computation.
- During the computation, the attacker can fault any intermediate value.
- A faulted value can be zero or random.
- The attacker can read the final result of the computation.

Fault injection — Definition

During the execution of an algorithm, the attacker can:

- modify any intermediate value by setting it to either a random value (*randomizing fault*) or zero (*zeroing fault*), such a fault can be either *permanent* or *transient*;
- skip any number of consecutive instructions (*skipping fault*).

At the end of the computation the attacker can read the result returned by the algorithm.

Attack order — Definition

We call *order* of the attack the number of fault injections in the computation.
An attack is said to be *high-order* if its order is strictly more than 1.

Data-Code Faulting Equivalence Lemma

Equivalence between faults on the code and on the data — Lemma

The effect of a skipping fault (i.e., fault on the code) can be captured by considering only randomizing and zeroing faults (i.e., fault on the data).

- If the skipped instructions are part of an arithmetic operation:
  - either the computation has not been done at all: its results becomes zero (if initialized) or random (if not),
  - or the computation has partly been done: its result is thus considered random at our modeling level.
- If the skipped instruction is a branching instruction, it is equivalent to fault the result of the branching condition:
  - at zero (i.e., `false`), to avoid branching,
  - at random (i.e., `true`), to force branching. □

Algorithm Description

- Low level enough for the attack to work if protections are not implemented.
- Intermediate variable that would appear during refinement could be the target of an attack, but such a fault would propagate to an intermediate variable of the high level description.

- Input:
  - A high level description of the computation, and
  - an attack success condition.
- Output:
  - Either the list of possible attacks, or
  - a proof that the computation is resistant to fault injections.

⇒ `http://pablo.rauzy.name/sensi/finja.html`

How does it Works?

- The description of the computation is transformed into a *term*.
- The term is a tree which encodes:
  - dependencies between the intermediate values, and
  - properties of the intermediate values (such as being null, being null modulo another term, or being a multiple of another term).
- Each intermediate value (subterms of the tree) can be faulted, in such case its properties become:
  - nothing, in the case of a randomizing fault, or
  - being null, in the case of a zeroing fault.
- Symbolic computation by term rewriting is used to simplify the term and the attack success condition.

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Rewriting System

- Most of the $\mathbb{Z}$ ring axioms,
- $\mathbb{Z}_N$ subrings,
- And a few theorems.

Rewriting System

- Most of the $\mathbb{Z}$ ring axioms:
  - neutral elements (0 for sums, 1 for products);
  - absorbing element (0, for products);
  - inverses and opposites;
  - associativity and commutativity;
  - but no distributivity (not confluent).
- $\mathbb{Z}_N$ subrings,
- And a few theorems.

Rewriting System

- Most of the $\mathbb{Z}$ ring axioms,
- $\mathbb{Z}_N$ subrings:
  - identity:
    - $(a \mod N) \mod N = a \mod N$,
    - $N^k \mod N = 0$;
  - inverse:
    - $(a \mod N) \times (a^{-1} \mod N) \mod N = 1$,
    - $(a \mod N) + (-a \mod N) \mod N = 0$;
  - associativity and commutativity:
    - $(b \mod N) + (a \mod N) \mod N = a + b \mod N$,
    - $(a \mod N) \times (b \mod N) \mod N = a \times b \mod N$;
  - subrings: $(a \mod N \times m) \mod N = a \mod N$.
- And a few theorems.

Rewriting System

- Most of the $\mathbb{Z}$ ring axioms,
- $\mathbb{Z}_N$ subrings,
- And a few theorems:
  - Fermat's little theorem;
  - its generalization, Euler's theorem;
  - Chinese remainder theorem;
  - Binomial theorem in $\mathbb{Z}_{r^2}$ rings
    $(1+r)^d \equiv 1 + dr \mod r^2$.

Testing Attacks

For each possible fault attack:

- the faulted term is simplified to propagate to modified properties;
- simplified terms (faulted and original) are then fed into the attack success condition;
- the attack success condition itself is then simplified to either true (the attack works) or false (it doesn't).

Minimal Example of Usage

- Computation: $t = a + b \times c$.
- Let's say the "attack" works if $t \not\equiv a \mod b$.

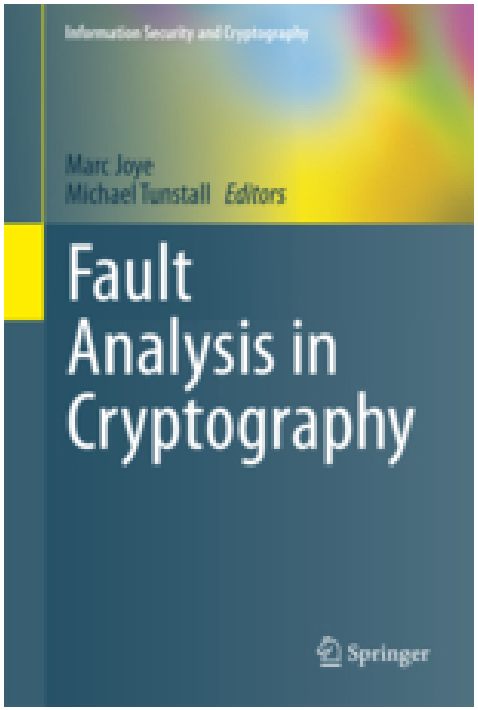
minimal-example.fia

```
noprop a, b, c ;

t := a + b * c ;

return t ;

%%

@ !=[b] a
```

- `finja minimal-example.fia -r`
- `finja minimal-example.fia -z`

- First formally proved fault injection attack countermeasures:
  - Aumüller et al., in [RG14a],
  - Vigilant, in [RG14b].
- Both countermeasures simplified.
- But both countermeasures are first-order.

- High-order attacks?
- High-order countermeasures?
- Proved high-order countermeasures?



FDTC
Fault Diagnosis and Tolerance in Cryptography

High-Order Attacks

- High-order attacks have been studied and shown practical:
  - *Fault Attacks for CRT Based RSA: New Attacks, New Results, and New Countermeasures* [KQ07],
    by C. H. Kim and J.-J. Quisquater at WISTP'07.
  - *Multi Fault Laser Attacks on Protected CRT-RSA* [TK10],
    by E. Trichina and R. Korkikyan at FDTC'10.

Existing High-Order Countermeasures?

- A few countermeasures claim to be second-order:
  - *Practical fault countermeasures for chinese remaindering based RSA* [CJ05],
    by M. Ciet and M. Joye at FDTC'05.
  - *On Second-Order Fault Analysis Resistance for CRT-RSA Implementations* [DGRS09],
    by E. Dottax, C. Giraud, M. Rivain, and Y. Sierra at WISTP'09.

  But they do not work in our more general fault model as our tool finja shows: `crt-rsa_ciet-joye.fia.zzt.html`, `crt-rsa_dottax-etal.fia.rzt.html`.
- We found no countermeasure claiming to resist > 2 faults.

- If we want a high-order countermeasure, we have to create it.
- What is a countermeasure?
- What makes a countermeasure work? What makes it fail?
- How do the existing first-order countermeasures work?

What is a Countermeasure?

- The goal of a countermeasure against fault-injection attacks is to avoid returning a compromised value to the attacker.
- This is done by *verifying the integrity of the computation* before returning its result, and returning a random number or an error constant rather than the actual result if appropriate.

Computation Integrity Verification

- Obvious idea: repeat the computation and compare the results.
- But of course that costs too much.
- Existing countermeasures optimize this idea in many different ways.

- What are the different methods used by the existing countermeasures to verify the computation integrity faster than $(M^d)^e \stackrel{?}{\equiv} M \mod N$?
- We used 4 main parameters to classify countermeasures.

1. Shamir's or Giraud's Family of Countermeasures

- Two main families of countermeasures:
  - descendants of Giraud's countermeasure [Gir06],
  - descendants of Shamir's countermeasure [Sha99].

Giraud's Family

- Use particular exponentiation algorithms.
- Keep track of variables involved in intermediate steps.
- Consistency check of an invariant that is supposed to be spread till the last steps.
- Examples of countermeasures in this family include:
  - Boscher et al. [BNP07],
  - Rivain [Riv09] (and its recently improved version [LRT14]),
  - Kim et al. [KKHH11].
- The detailed study of the countermeasures in Giraud's family is left as future work.

Shamir's Family

- Rely on a kind of "checksum" of the computation using smaller numbers:
  - RSA computes in rings $\mathbb{Z}_a$ where a is either a large prime number (e.g., $a = p$ or $a = q$) or the product of large prime numbers (e.g., $a = pq$).
  - Any small number b is coprime with a.
  - We have an isomorphism between the *overring* $\mathbb{Z}_{ab}$ and $\mathbb{Z}_a \times \mathbb{Z}_b$.
  - The nominal computation and the checksum can be conducted in parallel in $\mathbb{Z}_{ab}$.
- Attempt to assert that some invariants on the computations and the checksums hold.
- Many different ways to use the checksums and to verify these invariants.

Notation: $\mathbb{Z}_n$ is a shorthand for $\mathbb{Z}/n\mathbb{Z}$.

2. Test-Based or Infective Countermeasures

- A first way to classify countermeasures is to separate:
  - those which consist in step-wise internal checks during the CRT computation,
  - and those which use an infective computation strategy to make the result unusable by the attacker in case of fault injection.

Test-Based Countermeasures

Test-based countermeasure — Definition

A countermeasure is said to be *test-based* if it attempts to detect fault injections by verifying that some arithmetic invariants are respected, and branch to return an error instead of the numerical result of the algorithm in case of invariant violation.

- Examples of test-based countermeasures:
  - Shamir [Sha99],
  - Aumüller et al. [ABF+02],
  - Vigilant [Vig08],
  - Joye et al. [JPY01].

Infective Countermeasures

Infective countermeasure — Definition

A countermeasure is said to be *infective* if rather than testing arithmetic invariants it uses them to compute a neutral element of some arithmetic operation in a way that would not result in this neutral element if the invariant is violated.
It then uses the results of these computations to infect the result of the algorithm before returning it to make it unusable by the attacker (thus, it does not need branching instructions).

- Examples of infective countermeasures:
  - Blömer et al. [BOS03],
  - Ciet & Joye [CJ05],
  - Kim et al. [KKHH11].

Infection-Test Equivalence Property

Equivalence between test-based and infective verification — Proposition

Each test-based (resp. infective) countermeasure has a direct equivalent infective (resp. test-based) countermeasure.

- Invariants that must be verified by countermeasures are modular equality, i.e., they are of the form $a \stackrel{?}{\equiv} b \mod m$.
- Test-based: `if a != b [mod m] then return` *error*.
- Infective: `c := a - b + 1 mod m; ... return` S^c. □

- In our fault model, both the countermeasures claiming to be first-order and the ones claiming to be second-order actually offer the same level of protection.
  That is, they resist any number of randomizing faults, but can be broken by a well targeted fault injection + a skipping (test-based) or zeroing (infective) fault to bypass the right verification.

$\Rightarrow$ The concept of integrity verification does not depend on the attack order.

4. Usage of the Small Subrings

- In most countermeasures, the computations of S_p and S_q take place in overrings $\mathbb{Z}_{pr_1}$ and $\mathbb{Z}_{qr_2}$ rather than in $\mathbb{Z}_p$ and $\mathbb{Z}_q$.
- This allows the retrieval of the results modulo p and q, and verifying the signature modulo r_1 and r_2 (aforementioned checksums).

- Are the smaller rings used to verify the intermediate signatures?
- Or are they used directly to compute checksums that are verified?
- Does CRT recombination takes place in an overring?
- If r_1 is equal to r_2, what is permitted by the resulting symmetry?

Recap

Countermeasure	Family	Verification method/count	Intended order	Order	Small subrings usage
Shamir [Sha99]	Shamir	test / 1	1	0	$r_1 = r_2$, consistency of intermediate signatures
Joye et al. [JPY01]	Shamir	test / 2	1	0	checksums of the intermediate CRT signatures
Aumüller et al. [ABF$^+$02]	Shamir	test / 5	1	1	$r_1 = r_2$, consistency of the checksums of both intermediate signatures
Blömer et al. [BOS03]	Shamir	infection / 2	1	1	direct verification of the intermediate CRT signatures, CRT recombination happens in overring
Ciet & Joye [CJ05]	Shamir	infection / 2	2	1	checksums of the intermediate CRT signatures, CRT recombination happens in overring
Giraud [Gir06]	Giraud	test / 1	1	1	NA
Boscher et al. [BNP07]	Giraud	test / 1	1	1	NA
Vigilant [Vig08]	Shamir	test / 7	1	1	$r_1 = r_2$, embedded control values, CRT recombination happens in overring
Rivain [Riv09]	Giraud	test / 2	1	1	NA
Kim et al. [KKHH11]	Giraud	infection / 6	1	1	NA

Correctness of a countermeasure — Proposition

A countermeasure is correct if it verifies the integrity of

- the intermediate computation modulo p,
- the intermediate computation modulo q, and
- the CRT recombination (which can be subject to transient fault).

Additional verifications might be necessary if the computations needed for the countermeasure add new vulnerabilities.

- The straightforward countermeasure works at the arithmetic level.
- Any correct optimization of this algorithm is also a correct countermeasure.
- We saw that the countermeasures we studied are optimizations of the straightforward countermeasure. □

High-Order Countermeasures — Proposition

Against randomizing faults, all correct countermeasures are high-order.

However, there are no generic high-order countermeasures if the three types of faults in our attack model are taken into account, but it is possible to build nth-order countermeasures for any n.

- A random fault cannot induce a verification skip, whether test-based of infective.
- Repeating verifications n times can force the attacker to need $n + 1$ faults (one actually faulting the computation and the n others for bypassing the verifications). □

Correcting Shamir's Countermeasure

Algorithm: CRT-RSA with Shamir's countermeasure

Input: Message M, key (p, q, d, i_q) **Output:** Signature $M^d \mod N$, or error

1 Choose a small random integer r.

2 $p' = p \cdot r$

3 $q' = q \cdot r$

5 $S'_p = M^d \mod \varphi(p') \mod p'$ // Intermediate signature in $\mathbb{Z}_{pr}$

6 $S'_q = M^d \mod \varphi(q') \mod q'$ // Intermediate signature in $\mathbb{Z}_{qr}$

7 **if** $S'_p \not\equiv S'_q \mod r$ **then return** error

8 $S_p = S'_p \mod p$ // Retrieve intermediate signature in $\mathbb{Z}_p$

9 $S_q = S'_q \mod q$ // Retrieve intermediate signature in $\mathbb{Z}_q$

10 $S = S_q + q \cdot (i_q \cdot (S_p - S_q) \mod p)$ // Recombination in $\mathbb{Z}_N$

12 **return** S

Correcting Shamir's Countermeasure

Algorithm: CRT-RSA with Shamir's countermeasure

Input: Message M, key (p, q, d, i_q) **Output:** Signature $M^d \mod N$, or error

1 Choose a small random integer r.

2 $p' = p \cdot r$

3 $q' = q \cdot r$

4 **if** $p' \not\equiv 0 \mod p$ **or** $q' \not\equiv 0 \mod q$ **then return** error

5 $S'_p = M^d \mod \varphi(p') \mod p'$ // Intermediate signature in $\mathbb{Z}_{pr}$

6 $S'_q = M^d \mod \varphi(q') \mod q'$ // Intermediate signature in $\mathbb{Z}_{qr}$

7 **if** $S'_p \not\equiv S'_q \mod r$ **then return** error

8 $S_p = S'_p \mod p$ // Retrieve intermediate signature in $\mathbb{Z}_p$

9 $S_q = S'_q \mod q$ // Retrieve intermediate signature in $\mathbb{Z}_q$

10 $S = S_q + q \cdot (i_q \cdot (S_p - S_q) \mod p)$ // Recombination in $\mathbb{Z}_N$

11 **if** $S \not\equiv S'_p \mod p$ **or** $S \not\equiv S'_q \mod q$ **then return** error

12 **return** S

- We simplified Vigilant's countermeasure in 4 steps:
  - simplification of Coron et al.'s corrections [CGM+10]
    + our simplifications from our PPREW'14 paper [RG14b];
  - remove additional computation with random numbers;
  - taking advantage of Vigilant's clever sub-CRT embedding technique to verify the 3 necessary invariants in one single step in the small subring;
  - Bonus: transform the countermeasure to it's infective variant.

Algorithm: CRT-RSA with Vigilant's countermeasure

Input: Message M, key (p, q, d_p, d_q, i_q) **Output:** Signature $M^d \mod N$, or error

1 Choose a small random integer r, R_1, R_2, R_3, R_4. $N = p \cdot q$
2 $p' = p \cdot r^2$
3 $i_{pr} = p^{-1} \mod r^2$
4 $M_p = M \mod p'$
5 $B_p = p \cdot i_{pr}$; $A_p = 1 - B_p \mod p'$
6 $M'_p = A_p \cdot M_p + B_p \cdot (1 + r) \mod p'$ // CRT insertion of verification value in M'_p
7 $d'_p = d_p + R_3 \cdot (p - 1)$
8 $S'_p = {M'_p}^{d'_p \mod \varphi(p')} \mod p'$ // Intermediate signature in $\mathbb{Z}_{pr^2}$
9 **if** $M'_p \not\equiv M \mod p$ **or** $d'_p \not\equiv d_p \mod p - 1$ **or** $B_p \cdot S'_p \not\equiv B_p \cdot (1 + d'_p \cdot r) \mod p'$ **then return** error
10 $S_{pr} = S'_p - B_p \cdot (1 + d'_p \cdot r - R_1)$ // Verification value of S'_p swapped with R_1
11 $q' = q \cdot r^2$
12 $i_{qr} = q^{-1} \mod r^2$
13 $M_q = M \mod q'$
14 $B_q = q \cdot i_{qr}$; $A_q = 1 - B_q \mod q'$
15 $M'_q = A_q \cdot M_q + B_q \cdot (1 + r) \mod q'$ // CRT insertion of verification value in M'_q
16 $d'_q = d_q + R_4 \cdot (q - 1)$
17 $S'_q = {M'_q}^{d'_q \mod \varphi(q')} \mod q'$ // Intermediate signature in $\mathbb{Z}_{qr^2}$
18 **if** $M'_q \not\equiv M \mod q$ **or** $d'_q \not\equiv d_q \mod q - 1$ **or** $B_q \cdot S'_q \not\equiv B_q \cdot (1 + d'_q \cdot r) \mod q'$ **then return** error
19 $S_{qr} = S'_q - B_q \cdot (1 + d'_q \cdot r - R_2)$ // Verification value of S'_q swapped with R_2
20 **if** $M_p \not\equiv M_q \mod r^2$ **then return** error
21 $S_r = S_{qr} + q \cdot (i_q \cdot (S_{pr} - S_{qr}) \mod p')$ // Recombination checksum in $\mathbb{Z}_{Nr^2}$

23 **if** $N \cdot (S_r - R_2 - q \cdot i_q \cdot (R_1 - R_2)) \not\equiv 0 \mod Nr^2$ **then return** error
24 **if** $q \cdot i_q \not\equiv 1 \mod p$ **then return** error
25 **return** $S = S_r \mod N$ // Retrieve result in $\mathbb{Z}_N$

Algorithm: CRT-RSA with Vigilant's countermeasure

Input: Message M, key (p, q, d_p, d_q, i_q) **Output:** Signature $M^d \mod N$, or error

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2 $p' = p \cdot r^2$

3 $i_{pr} = p^{-1} \mod r^2$

4 $M_p = M \mod p'$

5 $B_p = p \cdot i_{pr}$; $A_p = 1 - B_p \mod p'$

6 $M'_p = A_p \cdot M_p + B_p \cdot (1 + r) \mod p'$ // CRT insertion of verification value in M'_p

7 $d'_p = d_p + R_3 \cdot (p - 1)$

8 $S'_p = {M'_p}^{d'_p \mod \varphi(p')} \mod p'$ // Intermediate signature in $\mathbb{Z}_{pr^2}$

9 **if** $M'_p \not\equiv M \mod p$ **or** $d'_p \not\equiv d_p \mod p - 1$ **or** $B_p \cdot S'_p \not\equiv B_p \cdot (1 + d'_p \cdot r) \mod p'$ **then return** error

10 $S_{pr} = S'_p - B_p \cdot (1 + d'_p \cdot r - R_1)$ // Verification value of S'_p swapped with R_1

11 $q' = q \cdot r^2$

12 $i_{qr} = q^{-1} \mod r^2$

13 $M_q = M \mod q'$

14 $B_q = q \cdot i_{qr}$; $A_q = 1 - B_q \mod q'$

15 $M'_q = A_q \cdot M_q + B_q \cdot (1 + r) \mod q'$ // CRT insertion of verification value in M'_q

16 $d'_q = d_q + R_4 \cdot (q - 1)$

17 $S'_q = {M'_q}^{d'_q \mod \varphi(q')} \mod q'$ // Intermediate signature in $\mathbb{Z}_{qr^2}$

18 **if** $M'_q \not\equiv M \mod q$ **or** $d'_q \not\equiv d_q \mod q - 1$ **or** $B_q \cdot S'_q \not\equiv B_q \cdot (1 + d'_q \cdot r) \mod q'$ **then return** error

19 $S_{qr} = S'_q - B_q \cdot (1 + d'_q \cdot r - R_2)$ // Verification value of S'_q swapped with R_2

21 $S_r = S_{qr} + q \cdot (i_q \cdot (S_{pr} - S_{qr}) \mod p')$ // Recombination checksum in $\mathbb{Z}_{Nr^2}$

23 **if** $pq \cdot (S_r - R_2 - q \cdot i_q \cdot (R_1 - R_2)) \not\equiv 0 \mod Nr^2$ **then return** error

25 **return** $S = S_r \mod N$ // Retrieve result in $\mathbb{Z}_N$

Algorithm: CRT-RSA with Vigilant's countermeasure

Input: Message M, key (p, q, d_p, d_q, i_q) **Output:** Signature $M^d \mod N$, or error

1 Choose a small random integer r, R_1, R_2. $N = p \cdot q$
2 $p' = p \cdot r^2$
3 $i_{pr} = p^{-1} \mod r^2$
4 $M_p = M \mod p'$
5 $B_p = p \cdot i_{pr}$; $A_p = 1 - B_p \mod p'$
6 $M'_p = A_p \cdot M_p + B_p \cdot (1 + r) \mod p'$ // CRT insertion of verification value in M'_p

8 $S'_p = {M'_p}^{d_p} \mod \varphi(p') \mod p'$ // Intermediate signature in $\mathbb{Z}_{pr^2}$
9 **if** $M'_p \not\equiv M \mod p$ **or** $B_p \cdot S'_p \not\equiv B_p \cdot (1 + d_p \cdot r) \mod p'$ **then return** error
10 $S_{pr} = S'_p - B_p \cdot (1 + d_p \cdot r - R_1)$ // Verification value of S'_p swapped with R_1
11 $q' = q \cdot r^2$
12 $i_{qr} = q^{-1} \mod r^2$
13 $M_q = M \mod q'$
14 $B_q = q \cdot i_{qr}$; $A_q = 1 - B_q \mod q'$
15 $M'_q = A_q \cdot M_q + B_q \cdot (1 + r) \mod q'$ // CRT insertion of verification value in M'_q

17 $S'_q = {M'_q}^{d_q} \mod \varphi(q') \mod q'$ // Intermediate signature in $\mathbb{Z}_{qr^2}$
18 **if** $M'_q \not\equiv M \mod q$ **or** $B_q \cdot S'_q \not\equiv B_q \cdot (1 + d_q \cdot r) \mod q'$ **then return** error
19 $S_{qr} = S'_q - B_q \cdot (1 + d_q \cdot r - R_2)$ // Verification value of S'_q swapped with R_2

21 $S_r = S_{qr} + q \cdot (i_q \cdot (S_{pr} - S_{qr}) \mod p')$ // Recombination checksum in $\mathbb{Z}_{Nr^2}$

23 **if** $pq \cdot (S_r - R_2 - q \cdot i_q \cdot (R_1 - R_2)) \not\equiv 0 \mod Nr^2$ **then return** error

25 **return** $S = S_r \mod N$ // Retrieve result in $\mathbb{Z}_N$

Algorithm: CRT-RSA with Vigilant's countermeasure

Input: Message M, key (p, q, d_p, d_q, i_q) **Output:** Signature $M^d \mod N$, or

1 Choose a small random integer r. $N = p \cdot q$

2 $p' = p \cdot r^2$

3 $i_{pr} = p^{-1} \mod r^2$

4 $M_p = M \mod p'$

5 $B_p = p \cdot i_{pr}$; $A_p = 1 - B_p \mod p'$

6 $M'_p = A_p \cdot M_p + B_p \cdot (1 + r) \mod p'$ // CRT insertion of verification value in M'_p

8 $S'_p = {M'_p}^{d_p} \mod \varphi(p') \mod p'$ // Intermediate signature in $\mathbb{Z}_{pr^2}$

9 **if** $M'_p + N \not\equiv M \mod p$ **then return** error

10 $S_{pr} = 1 + d_p \cdot r$ // Checksum in $\mathbb{Z}_{r^2}$ for S'_p

11 $q' = q \cdot r^2$

12 $i_{qr} = q^{-1} \mod r^2$

13 $M_q = M \mod q'$

14 $B_q = q \cdot i_{qr}$; $A_q = 1 - B_q \mod q'$

15 $M'_q = A_q \cdot M_q + B_q \cdot (1 + r) \mod q'$ // CRT insertion of verification value in M'_q

17 $S'_q = {M'_q}^{d_q} \mod \varphi(q') \mod q'$ // Intermediate signature in $\mathbb{Z}_{qr^2}$

18 **if** $M'_q + N \not\equiv M \mod q$ **then return** error

19 $S_{qr} = 1 + d_q \cdot r$ // Checksum in $\mathbb{Z}_{r^2}$ for S'_q

21 $S_r = S_{qr} + q \cdot (i_q \cdot (S_{pr} - S_{qr}) \mod p')$ // Recombination checksum in $\mathbb{Z}_{r^2}$

22 $S' = S'_q + q \cdot (i_q \cdot (S'_p - S'_q) \mod p')$ // Recombination in $\mathbb{Z}_{Nr^2}$

23 **if** $S' \not\equiv S_r \mod r^2$ **then return** error

25 **return** $S = S' \mod N$ // Retrieve result in $\mathbb{Z}_N$

Algorithm: CRT-RSA with Vigilant's countermeasure

Input: Message M, key (p, q, d_p, d_q, i_q) **Output:** Signature $M^d \mod N$, or a random value in $\mathbb{Z}_N$

1 Choose a small random integer r. $N = p \cdot q$
2 $p' = p \cdot r^2$
3 $i_{pr} = p^{-1} \mod r^2$
4 $M_p = M \mod p'$
5 $B_p = p \cdot i_{pr}$; $A_p = 1 - B_p \mod p'$
6 $M'_p = A_p \cdot M_p + B_p \cdot (1 + r) \mod p'$ // CRT insertion of verification value in M'_p

8 $S'_p = {M'_p}^{d_p} \mod \varphi(p') \mod p'$ // Intermediate signature in $\mathbb{Z}_{pr^2}$
9 $c_p = M'_p + N - M + 1 \mod p$
10 $S_{pr} = 1 + d_p \cdot r$ // Checksum in $\mathbb{Z}_{r^2}$ for S'_p
11 $q' = q \cdot r^2$
12 $i_{qr} = q^{-1} \mod r^2$
13 $M_q = M \mod q'$
14 $B_q = q \cdot i_{qr}$; $A_q = 1 - B_q \mod q'$
15 $M'_q = A_q \cdot M_q + B_q \cdot (1 + r) \mod q'$ // CRT insertion of verification value in M'_q

17 $S'_q = {M'_q}^{d_q} \mod \varphi(q') \mod q'$ // Intermediate signature in $\mathbb{Z}_{qr^2}$
18 $c_q = M'_q + N - M + 1 \mod q$
19 $S_{qr} = 1 + d_q \cdot r$ // Checksum in $\mathbb{Z}_{r^2}$ for S'_q

21 $S_r = S_{qr} + q \cdot (i_q \cdot (S_{pr} - S_{qr}) \mod p')$ // Recombination checksum in $\mathbb{Z}_{r^2}$
22 $S' = S'_q + q \cdot (i_q \cdot (S'_p - S'_q) \mod p')$ // Recombination in $\mathbb{Z}_{Nr^2}$
23 $c_S = S' - S_r + 1 \mod r^2$

25 **return** $S = S' {}^{c_p c_q c_S} \mod N$ // Retrieve result in $\mathbb{Z}_N$

Algorithm: Generation of CRT-RSA with Vigilant's countermeasure at order D

Input: order D **Output:** CRT-RSA algorithm protected against fault injection attack of order D

1 **print** Choose a small random integer r.
2 **print** $N = p \cdot q$
3 **print** $p' = p \cdot r^2$; $i_{pr} = p^{-1} \mod r^2$; $M_p = M \mod p'$; $B_p = p \cdot i_{pr}$; $A_p = 1 - B_p \mod p'$
4 **print** $M'_p = A_p \cdot M_p + B_p \cdot (1 + r) \mod p'$
5 **print** $q' = q \cdot r^2$; $i_{qr} = q^{-1} \mod r^2$; $M_q = M \mod q'$; $B_q = q \cdot i_{qr}$; $A_q = 1 - B_q \mod q'$
6 **print** $M'_q = A_q \cdot M_q + B_q \cdot (1 + r) \mod q'$
7 **print** $S'_p = {M'_p}^{d_p} \mod \varphi(p') \mod p'$
8 **print** $S'_q = {M'_q}^{d_q} \mod \varphi(q') \mod q'$
9 **print** $S_{pr} = 1 + d_p \cdot r$
10 **print** $S_{qr} = 1 + d_q \cdot r$
11 **print** $S_r = S_{qr} + q \cdot (i_q \cdot (S_{pr} - S_{qr}) \mod p')$
12 **print** $S' = S'_q + q \cdot (i_q \cdot (S'_p - S'_q) \mod p')$
13 **for** $i \leftarrow 1$ **to** D **do**
14 | **print** c_p; **print** i; **print** $= M'_p + N - M + 1 \mod p$
15 | **print** c_q; **print** i; **print** $= M'_q + N - M + 1 \mod q$
16 | **print** c_S; **print** i; **print** $= S' - S_r + 1 \mod r^2$
17 **end**
18 **print** $c^* =$
19 **for** $i \leftarrow 1$ **to** D **do**
20 | **print** c_p; **print** i; **print** $\times$
21 | **print** c_q; **print** i; **print** $\times$
22 | **print** c_S; **print** i; **print** $\times$
23 **end**
24 **print** 1
25 **print return** $S = S^{c^*} \mod N$

- Formal studies of these countermeasures allowed to understand their working factor.

→ We were able to fix the broken ones and to simplify many of them (e.g., original Vigilant's countermeasure: broken, 9 tests, 5 random numbers; our fixed and simplified version: working, 3 tests, 1 random number).

→ We were able to provide a recipe for high-order countermeasures.

- More importantly, the working factor is actually not tied to the BellCoRe attack, nor to the CRT-RSA algorithm.
- It is possible to abstract it and get a recipe for cost-effectively verifying the integrity of any arithmetic computation.

- Idea: verify the integrity of the computation by introducing redundancy.
- Simply repeating the computation and comparing results is bad:
  (a) it is too expensive, and
  (b) nothing stops the attacker from injecting the same fault twice.
- Thus, existing countermeasures optimize this idea in different ways.

- The *entanglement* protection scheme solves both issues, by:
  - lifting the computation to an over-structure (a direct product) allowing (*a*) to project the result back onto the original structure, and (*b*) to project a checksum onto a smaller structure (e.g., `int32`-sized);
  - performing in parallel the same computation is the smaller structure;
  - both the checksum and the smaller result should be equal.
- The redundant part of the computation is almost free (arithmetic with 32-bit vs. 2,048-bit numbers).
- It is very hard to precisely fault the small computation to produce a consistent value modification.
- Limitation: possible collisions in the small structure.
  Mitigated by the possibility to use several different small structures.

- At IMDEA Software Institute (Madrid, Spain), I developed a compiler called enredo, while supervised by Gilles Barthe, François Dupressoir and Pierre-Yves Strub.
- Automated insertion of the *entanglement* countermeasure into arbitrary code.

⇒ `http://pablo.rauzy.name/sensi/enredo.html`

- Short demo.

- We already have:
  - an executable code output (Python),
  - a correctness proof of the code transformation.
- Benchmark of the cost of the countermeasure.
- Security proof.
- Protected implementations of currently unprotected algorithms.
- Practical lab tests.

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That was it. Questions?

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